DETERMINATION OF EROS PHYSICAL PARAMETERS FOR NEAR EARTH ASTEROID RENDEZVOUS ORBIT PHASE NAVIGATION

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Navigation of the orbit phase of the Near Earth Asteroid Rendezvous (NEAR) mission will require determination of certain physical parameters describing the size, shape, gravity field, attitude and inertial properties of Eros. Prior to launch, little was known about Eros except for its orbit which could be determined with high precision from ground based telescope observations. Radar bounce and light curve data provided a rough estimate of Eros shape and a fairly good estimate of the pole, prime meridian and spin rate. The determination of the NEAR spacecraft orbit requires a high precision model of Eros's physical parameters and the ground based data provides only marginal *a priori* information. The initial orbit determination strategy is therefore concerned with developing a precise model of Eros.

On December 23, 1998, the NEAR spacecraft flew by Eros on a high velocity trajectory that provided a brief glimpse and allowed for a crude estimate of the pole, prime meridian and mass of Eros. Estimates of Eros physical parameters obtained from this flyby are presented. Application of this new knowledge to simplification of Eros orbital operations and the orbit determination strategy will be discussed.

INTRODUCTION

The original plan for Eros orbital operations was to execute a series of rendezvous burns beginning on December 20,1998 and insert into a close Eros orbit in January 1999. As a result of an unplanned termination of the first rendezvous burn, the NEAR spacecraft continued on its high velocity approach trajectory and passed within 3900 km of Eros on December 23, 1998. The planned rendezvous burn was delayed until January 3, 1999 which resulted in the spacecraft being placed on a trajectory that slowly returns to Eros with a subsequent delay of close Eros orbital operations until February 2000. The flyby of Eros provided a brief glimpse and allowed for a crude estimate of the pole, prime meridian and mass of Eros. More important for navigation, orbit determination software was executed in the landmark tracking mode to determine the spacecraft orbit and a preliminary shape and landmark data base has been obtained. The flyby also provided an opportunity to test orbit determination operational procedures that will be used for orbital operations.

The initial attitude and spin rate of Eros, as well as estimates of reference landmark locations, are obtained from images of the asteroid. These initial estimates are used as *a priori* values for a more precise refinement of these parameters by the orbit determination software which combines optical measurements with Doppler tracking and laser altimetry data to obtain solutions for the required parameters. As the spacecraft is maneuvered closer to the asteroid, estimates of spacecraft state, asteroid attitude, solar pressure, landmark locations and Eros physical parameters including mass, moments of inertia and gravity harmonics are determined with increasing precision.

Gravity harmonics are in themselves of interest to science. When compared with the asteroid shape, some insight may be obtained into Eros' internal structure. The location of the center of mass derived

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from the first degree harmonic coefficients give a direct indication of overall mass distribution. The second degree harmonic coefficients relate to the radial distribution of mass. Higher degree harmonics may be compared with surface features to gain additional insight into mass distribution.

The elements of the inertia tensor are also of interest. The moments of inertia about the principal axes provide insight into the radial mass distribution. Determination of the principal axes moments of inertia will depend on observing free precession in the asteroid's attitude dynamics.

EROS FLYBY NAVIGATION OPERATIONS

For the nominal mission, navigation operations were separated into two distinct mission phases being interplanetary and Eros orbital. Interplanetary navigation involves long periods of collecting data punctuated by short bursts of activity related to planetary or asteroid encounters. The orbit determination is performed on long arcs of Doppler and range data and is primarily Earth based. The spacecraft is effectively observed from the Earth on a star background. Close to the Mathilde encounter of June 1997, the radiometric tracking data was aided by optical images of Mathilde taken during approach. During Eros orbital operations, the orbit determination is primarily Eros based and relies on the gravitational acceleration of Eros and the resultant orbital dynamics. Here, optical images of landmarks permit high precision orbits to be determined.

During the flyby of Eros on December 23, 1998, the navigation operations did not fall into either of the categories defined above. The gravitational acceleration of Eros was barely detected by the Doppler data and landmark images were of such low resolution as to be barely useful for orbit determination. During the nominal planned mission, the spacecraft would fly through the transition between the interplanetary and orbital mission phases in a few days. Thus, it was possible to optimize the navigation system by treating these two phases as separate and distinct. During the unplanned flyby, the spacecraft was in this transition zone when all the useful data was obtained and because of the high spacecraft velocity, this period was only a few hours in duration. Fortunately, after the encounter, the navigation team had several months to revise the orbit determination strategy and analyze the data. The result was a revision of the Eros model parameters and a vast improvement of our knowledge of Eros over Earth based observations that will simplify the orbital operations beginning in February 2000.

Orbit Determination Strategy

As the spacecraft flew by Eros at a velocity of about 1 km/s the interplanetary navigation strategy was used for determining the orbit and computing maneuvers. At this time no effort was made to track surface features which were barely discernible in the MSI images. Doppler, range and optical data was cataloged and saved for post encounter analysis. The post encounter orbit determination strategy evolved directly from the Eros orbit phase strategy. Some refinements were necessary in order to separate the desired information from the noise. The data types were Doppler and range data obtained from the Deep Space Network (DSN) and landmark line and pixel pairs from the Near Multispectral Imager(MSI). In addition, light curve data was obtained from the measured brightness of Eros during approach and images of Eros were used to track the center of brightness and improve the Eros relative spacecraft ephemeris.

Eros *a priori* Physical Model

Determination of the spacecraft orbit about Eros is intimately associated with the development of an accurate physical model of Eros. Eros is the principal source of perturbations of the spacecraft's trajectory and the principal source of data for determining the orbit. The model of Eros used for orbit determination will be similar to the model used by the science team. The major difference is in emphasis of detail.

During a particularly close Earth approach (0.15 AU) in January 1975, there was a coordinated ground-based observation campaign to characterize the physical nature of Eros. Photometric, spectroscopic and radar measurements provided a diverse data set that allowed the asteroid's size, shape and spectral type to be determined. The asteroid's shape was approximated as a triaxial ellipsoid with dimensions 40.5 km x 14.5 km x 14.1 km and with a north rotation pole position (1950) given by an ecliptic longitude and latitude of 16° and 11° respectively. Eros is an S type object with a geometric albedo of 0.16. From the light curve variations, which reach 1.47 magnitude in brightness, the rotation period has been determined as 5.27011 hours. The absolute magnitude of Eros (at zero phase angle and one AU from both the sun and Earth) is 11.16.

The observed approximate shape has been embellished with craters and surface features by the Applied Physics laboratory (APL) to define a reference model for navigation and mission design studies. This model is described by 4,202 vertices that are covered by 8,400 triangular plates. The parameters of the APL plate model are given in Table 1. For rotational stability, the polar axis or z-axis is perpendicular to the long axis and is the principal axis of inertia with the greatest moment of inertia. The long axis is therefore in the equatorial plane and is taken to be the x-axis and is the axis with minimum value for the moment of inertia. The y-axis completes the right hand body fixed coordinate system and is the axis with intermediate value for the moment of inertia or the unstable axis. Longitude (body-centered) is measured positive east from the x-axis. The mass properties and gravity harmonics given in Table 5 were obtained by numerical integration over the volume enclosed by the APL plate model assuming a constant density of 3.5 g/cm^3.

Table 1
A PRIORI PHYSICAL MODEL OF EROS

Parameters	Values			
<u>Size and Shape</u>				
volume	3,790 km^3			
semi x-axis, y-axis, z-axis	16.7 km	8.6 km	6.3 km	
<u>Mass properties</u>				
density	3.5 g/cm^3			
mass	1.3×10^{16} kg			
GM	8.86×10^{-4} km^3/s^2			
I_{xx}, I_{yy}, I_{zz}	22.9 km^2	63.9 km^2	70.9 km^2	
I_{xy}, I_{xz}, I_{yz}	0	0	0	
<u>Pole</u>				
right ascension	16.4 deg			
declination	15.2 deg			
prime meridian	344.0 deg (J2000)			
<u>Gravity harmonics</u>				
r_0	16.0 km			
C_{20}, C_{22}	-3.0×10^{-2}	$+3.8\times10^{-2}$		
C_{40}, C_{42}, C_{44}	$+4.1\times10^{-3}$	-6.2×10^{-3}	$+5.1\times10^{-3}$	
C_{60}, C_{62}, C_{64}, C_{66}	-7.4×10^{-4}	$+9.9\times10^{-4}$	-8.7×10^{-4}	$+8.4\times10^{-4}$

DETERMINATION OF EROS PHYSICAL PARAMETERS

The procedure for determining Eros physical parameters involved processing the available data in a sequence of orbit determination solutions. The first step was to process light curve data to determine the prime meridian and verify the spin rate determined from Earth based telescope observations of Eros. The next step was to identify a crater on Eros that the prime meridian is defined to pass through and locate the center of mass which was assumed to coincide with the center of brightness. All of the useable images were inspected and line pixel pairs for a dozen or so landmarks were determined. This data was used to prepare a picture sequence file for processing by orbit determination software.

Orbit determination solutions were obtained for landmark locations that were compared with a shape model that was developed by Cornell from the images. The Cornell shape model was stretched, translated and rotated with respect to the assumed center of mass to match the landmark locations. A final joint solution for all the estimated parameters was obtained that included Eros mass, landmark locations and the spacecraft ephemeris. The final shape model was numerically integrated assuming constant density to obtain an Eros gravity field and inertia tensor.

Eros Attitude Determination

Since the determination of Eros attitude by an orbit determination filter is an iterative process, an initial guess is needed to get the process started. This initial guess need not be accurate but must at least place Eros in the correct quadrant. The pole is known form Earth based measurements, as described above, to about 5 degrees and the spin rate is known to less than one percent. Assuming principal axis rotation, the only other attitude parameter that is unknown is the location of the prime meridian in inertial space. The assumption of rotation about principal axes was critical to initializing the filter. If the wobble of Eros was more than about 5 degrees, a rather complicated procedure would have had to be executed in order to converge the orbit determination solution. Even though prior observations were able to determine an accurate spin rate, these observations were not accurate enough to predict the location of the prime meridian at the time of the flyby. Several acquisitions of light curves were scheduled on approach to and departure from Eros that enabled the prime meridian to be determined with sufficient accuracy to serve as *apriori* for orbit determination. There remained an ambiguity of 180 degrees that would be resolved later depending on which end of Eros is selected to define zero longitude.

During approach to Eros, useable light curve data was obtained on November 11 and December 15, 1998 and after the flyby on April 14, 1999. At these times Eros illuminated one pixel and was too far away to be useful for landmark tracking. However, the variation in brightness resulting from its rotation provided a light curve that could be analyzed. The amplitude, frequency and phase of the light curve could be related to Eros's shape and attitude. Light curves taken from different directions and sun lighting conditions could be processed to determine much useful information about Eros. Given the shape of Eros, the spin rate and prime meridian angle W could be observed directly in the frequency and phase. At first these parameters were estimated assuming Eros was a triaxial ellipsoid with the dimensions given in Table 1. After the flyby, a more detailed model was developed by Cornell and used to generate a simulated light curve. The simulated light curve was made from simulated images of Eros constructed from the model with the correct trajectory determined after the flyby. The November 11, 1998 light curve is shown on Figure 1 and the April 14, 1999 light curve is shown on Figure 2. The solid curve was obtained by processing simulated images to determine the total brightness. The amplitude was scaled to fit the observed data points which were obtained by processing MSI images. The observed brightness of Eros fit the simulated light curve remarkably well. The high frequency variation in brightness may be attributed to reflections from craters that were not modeled in the simulated light curve. The Eros spin rate was determined by observing the minimums on November 19, 1998 shown on Figure 1 and determining the angle W at these times from the orbit determination solution. The orbit determination solution was mapped ahead to April 14, 1999 and another determination of the angle W was made at the times of the minimums shown on Figure 2. The Eros spin rate was determined by simply dividing the total change in W between these two epochs by the elapsed time. The period of Eros determined from the rotation rate is 5.270371 ($\pm 0.5 \times 10^{-04}$) hr and will permit a prediction of the prime meridian to within a few degrees during the return to Eros.

Landmark Location and Shape Model

A powerful data type for determining the NEAR spacecraft trajectory and rotation of Eros is optical tracking of landmarks. Tracking individual landmarks, which are craters of various sizes, enables orbit determination accuracies on the order of the camera resolution or several meters. This far exceeds the accuracy that can be obtained by fitting limb data or by any measurement

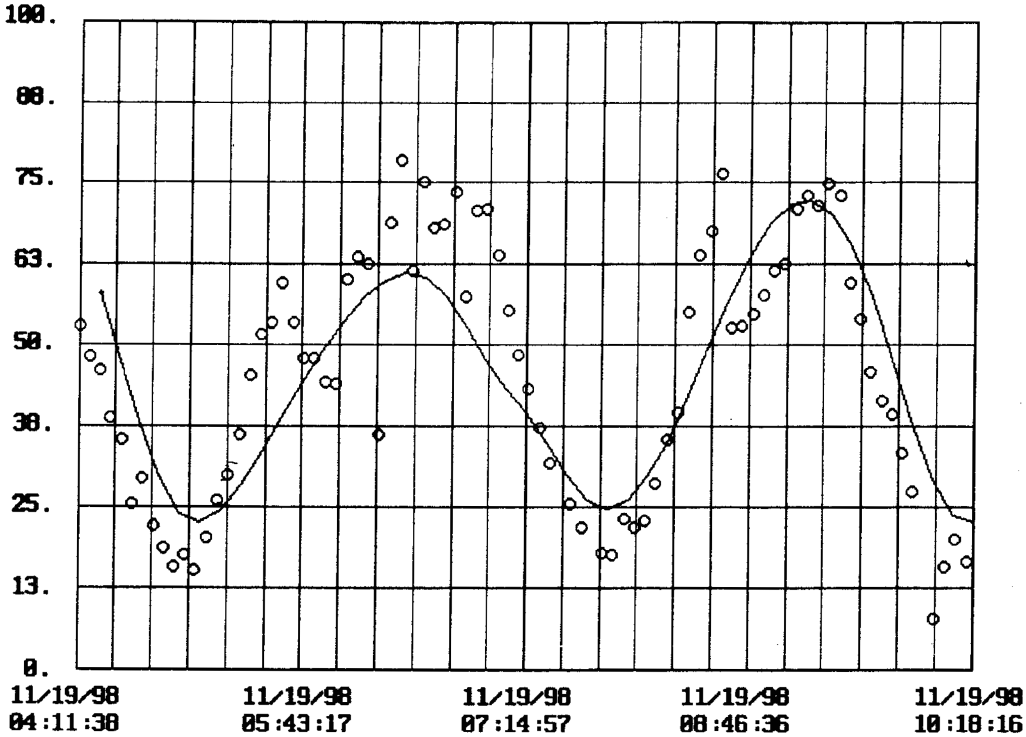

Figure 1 November 11, 1998 Light Curve

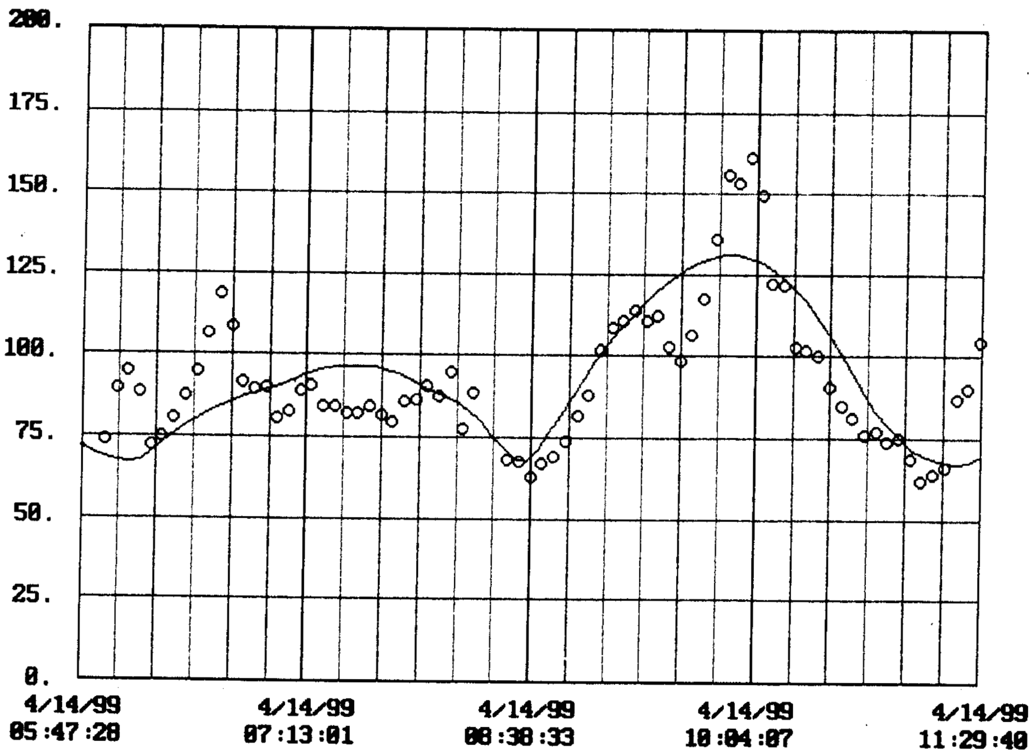

Figure 2 April 14,1999 Light Curve

scheme that is dependent on developing a precise shape model. We need only develop a data base of landmarks and identify the landmarks on more than one image in order to obtain useful information about the spacecraft orbit or Eros's rotation. The problem of identifying and cataloging landmarks is aided by referring the landmarks to a model of the topographic surface or shape model.

Most of the landmark tracking images were taken a few hours before closest approach where the lighting was optimal. A total of 12 navigation landmarks were identified is a set of 15 MSI optical navigation images. On the average, about half the landmarks on each image and the landmark data set therefore consists of about 90 line and pixel pairs. These landmark image data were weighted using a 0.5 pixel noise value. From this same set of images, it is possible to construct a shape model of the Eros surface. It can be shown from purely geometrical considerations that a shape model may be developed from stereoscopic images if one knows only the focal length of the camera and the range to the object. Thus, a fairly accurate shape model may be developed from the images without a knowledge of the spacecraft orbit or Eros attitude. A model was developed at Cornell by Peter Thomas and is being used as an aid in identifying landmarks. A further use for this model is the development of a gravity field and this will be discussed below.

Figure 3 shows an actual image obtained during the flyby of Eros and a simulated image from the shape model showing the same view of Eros. The simulated image shows the same general overall shape and is therefore useful for identifying an area on Eros to begin searching for landmarks. In the actual image, the large crater at the bottom was used to define zero longitude. Thus the x axis of Eros is along the long dimension of Eros and the y axis projects out of the large bright area to the right. The spin axis, or z axis, projects out of the image and to the left. The north pole of Eros is just below and to the left of the bright area in the images. Even though much information about the size and rotation of Eros can be seen in this image, particularly in the context of the other 15 images, only the line and pixel location of the large crater and several smaller craters, that can be seen under closer examination, are input to the orbit determination filter. All the navigation information useful for orbit determination is obtained by observing the rotation of the 12 landmarks that were identified.

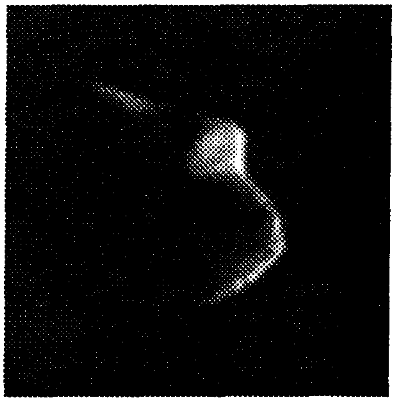

Figure 3 Eros Actual and Simulated Images

Orbit Determination Solution

The first step for obtaining an orbit determination solution is to assemble the data in files in a format that is convenient for the orbit determination software to process. Doppler and range data are assembled into a tracking data file that contains the measurements and calibration data. The calibration data includes constants and polynomial input for troposphere and ionosphere models, a solar plasma model, a solid Earth tide model and continental drift as well as the DSN station locations. A landmark file consisting of *a priori* landmark locations and a unique identification number is assembled along with a picture sequence file that contains spacecraft attitude, camera alignment and line-pixel pairs for each landmark that is identified. Additional files that are needed include the spacecraft clock, a solar pressure model, propulsive maneuvers and a file of initial state vectors including the spacecraft, planets and Eros. Finally, a file of ancillary data including numerical integration controls, estimated parameters and program controls is needed.

The estimated parameters effectively define the orbit determination solution strategy when the appropriate statistical constraints are placed on the *a priori* values of these parameters and the length of the data arc or data to be included in the solution is defined. The estimated parameters include spacecraft state, Eros ephemeris, Eros attitude and spin, Eros gravity, landmark locations, two propulsive maneuvers, solar radiation pressure and three components of stochastic acceleration. An important modification of the orbit phase orbit determination strategy was necessary in order to solve for landmark locations. In the orbit phase, the mass of Eros presents a strong signature in the Doppler data that permits a high precision determination of mass and the center of mass. Since the center of mass of Eros is the center of the coordinate system used for formulating the equations of motion, it is necessary to determine where it is relative to the spacecraft before we can solve for landmark locations. During the Eros flyby, it was not possible to make this determination because the gravity signature was too weak. In order to tie the landmark locations to a center, an artificial landmark was introduced with coordinates of zero. A single image was selected that showed Eros in maximum extension (Figure 3) and the location of the center of mass was defined to be at the optical center of this image. It is estimated that this introduced a systematic error of about one kilometer in the landmark coordinate estimation. In order to prevent the Eros body fixed coordinates from migrating, the Eros spin vector x and y coordinates ω_x and ω_y are constrained to be zero and the y component of the longitude reference landmark is also constrained to be zero. This strategy forces the spin vector in the same direction as Eros pole or z axis and forces the longitude reference landmark to be in the $y - z$ plane and thus be on the prime meridian with a longitude of zero degrees.

The data arc begins on November 25, 1998 and continues through January 12, 1999. The data was processed by a Square Root Information Filter (SRIF) with stochastic noise introduced to account for the random solar pressure disturbance caused by attitude maneuvers conducted by the NEAR spacecraft during the flyby. The stochastic acceleration had a magnitude of $2 \times 10^{-12}\text{km}\backslash\text{s}^2$ and correlation time of two days. A plot of the Doppler residuals is shown on Figure 6. The data arc contains two maneuvers on December 20, 1998 and January 3, 1999. Each of these maneuvers were modeled as two impulses that span the finite motor burn. The effect of this solution strategy is to uncouple the solution for Eros physical parameters from the data taken before the first of these maneuvers and after the second. Thus, the affect of execution errors associated with these maneuvers is minimized. The long arc of range and Doppler data between these maneuvers is useful for determining the Eros ephemeris and the spacecraft velocity.

The solution for Eros mass is dependent on the ability of the filter to resolve the acceleration of Eros during the flyby and the total change in velocity. Because of the effect of non gravitational accelerations, the detection of the acceleration during the flyby is the stronger of the two determining factors. The observed change in spacecraft velocity is masked by these accelerations or other error sources that appear as an acceleration. Since the Eros mass determination is particularly important for designing the NEAR orbit phase trajectory, it may be useful to develop a more positive indication that the mass is observable in the data other than the solution and covariance generated by the orbit determination filter. Figure 7 shows a plot of Doppler residuals with no Eros mass. The Eros mass is suppressed in both the model and in the solution. The assumption is that Eros has no mass. A clear peaking of the Doppler residuals is evident in Figure 7 at the time of the flyby. A signature of about 7 mHz is apparent in the data. Figure 8 shows the same segment of data for the solution including Eros's mass. The mass signature is flattened some but considerable unexplained residual signature remains. This is consistent with the solution and error obtained for Eros's mass. The formal mass solution in terms of the gravitational parameter is $4.8(\pm 1.2) \times 10^{-4}\text{km}^3/\text{s}^2$. The error shown in parenthesis is one sigma. The corresponding mass is then 7.2×10^{18} grams.

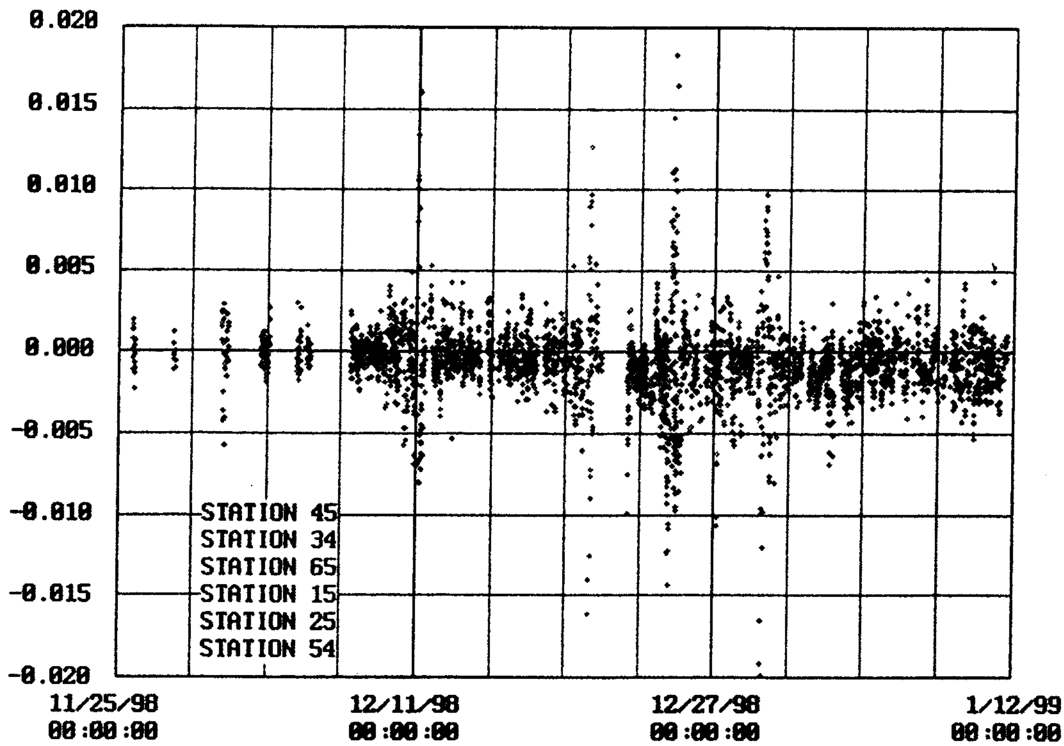

Figure 4 Eros Flyby Doppler Residuals

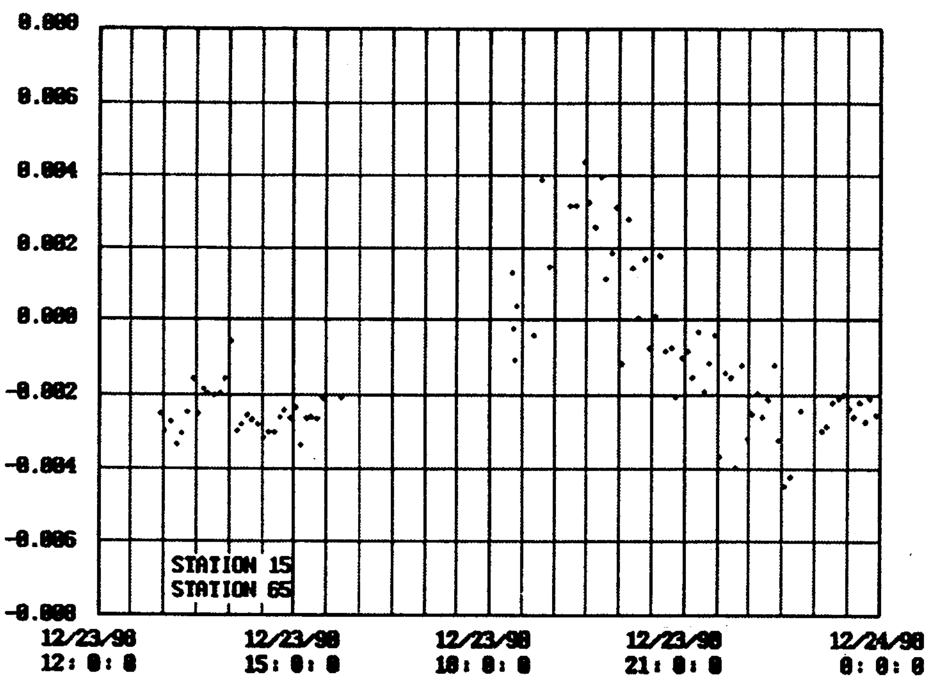

Figure 5 Eros Flyby Doppler Residuals Omitting Eros Mass

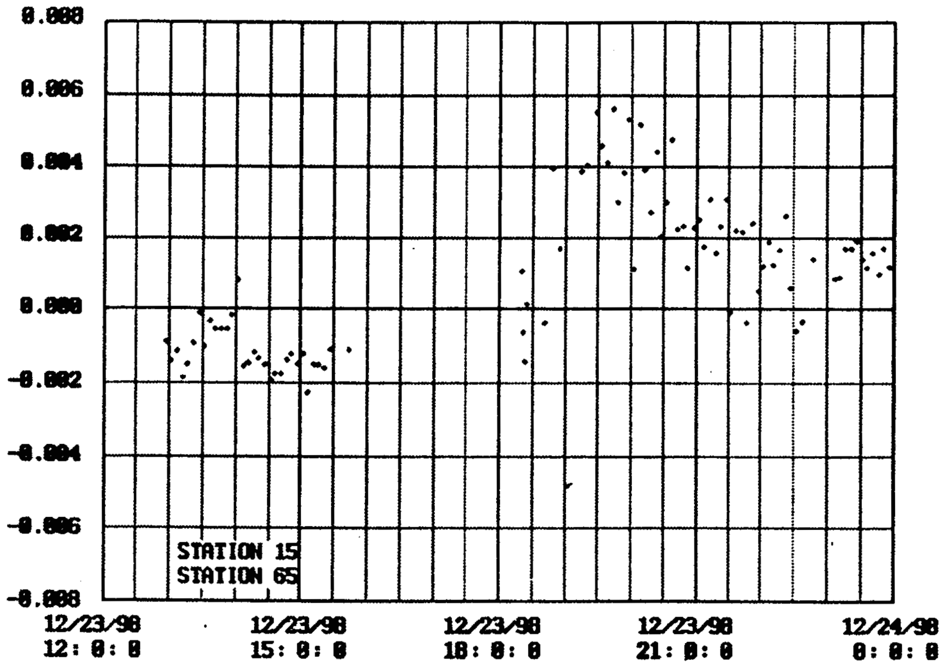

Figure 6 Eros Flyby Doppler Residuals Including Eros Mass

The solution for Eros physical parameters is summarized in Table 2. This solution compares well with the *a priori* values given in Table 1. Particularly important is the solution for the pole. The right ascension of 15.6($\pm$3.7) deg and declination of 16.4($\pm$1.8)deg places the pole in a favorable direction relative to the Eros orbit plane such that the preflight mission design largely remains valid. The absence of any significant wobble simplifies navigation operations during the orbit phase. The prime meridian angle and rotation rate determination will enable plans for imaging Eros during approach to be made in advance of arrival.

POST FIT DATA ANALYSIS

The solution for Eros rotational motion and physical parameters obtained from spacecraft orbit determination software is in the form of parameters that may not be useful for further data processing or data analysis. The rotational motion is in the form of a file of attitude as a function of time, the shape model consists of about 4000 triangular plates that are pieced together to form Eros's shape and the orbit determination process did not produce gravity harmonic coefficients that would be needed as *a priori* upon return to Eros. Further data processing is required to understand the forced precession of Eros from the solar tide, to develop shape harmonic coefficients that are needed for processing NLR altimetry data and to generate gravity harmonic coefficients.

Polar Motion

An important result that may be obtained from the NEAR data is an estimate of the moments of inertia about the principal axes. The moments of inertia provide insight into the radial distribution of mass. Estimates of the moments of inertia cannot be obtained if Eros is in principal axes rotation and there is no observed precession. Therefore, one of the priorities of the NEAR mission is to measure the precession of Eros. Precession results from disturbances of Eros rotational motion from quakes, meteorite impacts or gravitational accelerations. The free precession resulting from distinct events will damp out depending on the rate of energy dissipation. The forced precession from external gravity sources persist but are low in amplitude. The Suns gravity gradient results in a small forced precession that may be detected over several years but in any event must be accounted for in attempting to observe free precession that remains from past events.

Table 2
SOLUTION FOR EROS PHYSICAL PARAMETERS

Parameters	Values			
<u>Size and Shape</u>				
volume	3,790 km^3			
semi x-axis, y-axis, z-axis	16.7 km	8.6 km	6.3 km	
<u>Mass properties</u>				
density	3.5 g/cm^3			
mass	1.3×10^{16} kg			
GM	8.86×10^{-4} km^3/s^2			
I_{xx}, I_{yy}, I_{zz}	22.9 km^2	63.9 km^2	70.9 km^2	
I_{xy}, I_{xz}, I_{yz}	0	0	0	
<u>Pole</u>				
right ascension	16.4 deg			
declination	15.2 deg			
prime meridian	344.0 deg (J2000)			
<u>Gravity harmonics</u>				
r_0	16.0 km			
C_{20}, C_{22}	-3.0×10^{-2}	$+3.8\times10^{-2}$		
C_{40}, C_{42}, C_{44}	$+4.1\times10^{-3}$	-6.2×10^{-3}	$+5.1\times10^{-3}$	
C_{60}, C_{62}, C_{64}, C_{66}	-7.4×10^{-4}	$+9.9\times10^{-4}$	-8.7×10^{-4}	$+8.4\times10^{-4}$

As Eros orbits the Sun, it is subjected to a torque from the gradient of the Sun's gravity known as the solar tide. While this torque is small, it is conservative when averaged over many revolutions and does not add net rotational kinetic energy. However, it does result in a significant precession of the angular momentum vector. The force of the Sun on an elementary mass element of Eros is given by

$$\mathbf{dF} = \frac{\mu_s}{|\mathbf{r_s} - \mathbf{r}|^3}(\mathbf{r_s} - \mathbf{r})\mathrm{dm} \tag{1}$$

where $\mathbf{r_s}$ is the vector from the center of Eros to the Sun and $\mathbf{r}$ is the vector to an elementary mass element. The total applied moment to the asteroid is obtained by integrating the force times the moment arm over the density and volume of Eros.

$$\mathbf{M} = \iiint_V \left(\mathbf{r} \times \frac{\mathbf{dF}}{\mathrm{dm}}\right) \rho(\mathrm{r}, \lambda, \phi)\mathrm{dV} \tag{2}$$

Substituting the gravity force into the moment equation we obtain the result given by Greenwood[5] and others.

$$\mathbf{M} = \mu_s \iiint_V \frac{\mathbf{r} \times (\mathbf{r_s} - \mathbf{r})}{|\mathbf{r_s} - \mathbf{r}|^3} \rho(r, \lambda, \phi) dV \tag{3}$$

The distance from the Sun to the mass elements may be approximated by projecting the location of the mass elements onto the Eros-Sun vector ignoring parallax,

$$|\mathbf{r_s} - \mathbf{r}| = r_s - \frac{\mathbf{r_s} \cdot \mathbf{r}}{\mathbf{r_s}}$$

and the required inverse cube may be approximated by the first two terms of the Taylor series

$$\frac{1}{|\mathbf{r_s} - \mathbf{r}|^3} = \frac{1}{r_s^3}\left[1 + 3\frac{\mathbf{r_s} \cdot \mathbf{r}}{r_s^2}\right]$$

Replacing the vectors by components we obtain

$$\mathbf{M} = \frac{\mu}{r_s^5} \iiint_V \begin{bmatrix} yz_s - zy_s \\ zx_s - xz_s \\ xy_s - yx_s \end{bmatrix} \left[r_s^2 + 3(xx_s + yy_s + zz_s)\right] \rho(r, \lambda, \phi) dV \tag{4}$$

Since the origin of the coordinate system is the center of mass, the first order terms in x, y, and z integrate to zero and the second order terms integrate to moments and products of inertia.

$$\mathbf{M} = \frac{3\mu}{r_s^5} \begin{bmatrix} y_s z_s (I_{zz} - I_{yy}) + (y_s^2 - z_s^2) I_{yz} - x_s z_s I_{xy} + x_s y_s I_{xz} \\ x_s z_s (I_{xx} - I_{zz}) + (z_s^2 - x_s^2) I_{xz} - x_s y_s I_{yz} + y_s z_s I_{xy} \\ x_s y_s (I_{yy} - I_{xx}) + (x_s^2 - y_s^2) I_{xy} - y_s z_s I_{xz} + x_s z_s I_{yz} \end{bmatrix} \tag{5}$$

A Cartesian coordinate system is defined with the z axis in a direction normal to the Eros orbit plane. The $x - y$ plane is the Eros orbit plane and x points toward the Sun. The Cartesian coordinate frame (x_s, y_s, z_s) is defined by rotating through the angles ϕ and θ respectively. The z_s axis is aligned along the Eros spin vector or north pole and the $x_s - y_s$ plane is the Eros equatorial plane. The angle ϕ is the true anomaly of Eros in its orbit about the Sun and θ is the angle between z and z_s. We thus have

$$x_s = r_s \sin\theta \cos\phi$$

$$y_s = -r_s \sin\phi$$

$$z_s = r_s \cos\theta \cos\phi$$

The coordinate frame defined above can be made effectively into a body fixed frame by averaging the torque over one complete revolution and treating Eros as thought it were symmetrical about the spin axis. The gravity gradient moment equations for principal axes becomes

$$\mathbf{M} = \frac{3\mu}{r_s^3} \begin{bmatrix} -\cos\theta \sin\phi \cos\phi (I_{zz} - \frac{I_{xx} + I_{yy}}{2}) \\ -\sin\theta \cos\theta \cos^2\phi (I_{zz} - \frac{I_{xx} + I_{yy}}{2}) \\ 0 \end{bmatrix} \tag{6}$$

Euler's equation relates the applied moment to the rotational motion.

$$\mathbf{M} = I\dot{\mathbf{\Omega}} + \mathbf{\Omega} \times \mathbf{H} \tag{7}$$

where the angular momentum is given by

$$\mathbf{H} = I\,\mathbf{\Omega} \tag{8}$$

A complete solution of Euler's equation is an imposing task. In order to get a solution for the long term precession and nutation, we may make the assumption that the angular acceleration ($\dot{\mathbf{\Omega}}$) is zero. From Euler's equation, we thus have

$$\mathbf{M} = \begin{bmatrix} \omega_y H_z \\ \\ \omega_x H_z \\ \\ 0 \end{bmatrix} \tag{9}$$

$$H_z = I_{zz}\omega_z \tag{10}$$

an the forced precession($\dot{\Psi}$) and nutation ($\dot{\eta}$) arising from Eros orbital motion is given by

$$\omega_x = \dot{\Psi}\sin\theta \tag{11}$$

$$\omega_y = \dot{\eta} \tag{12}$$

Solving for precession and nutation we obtain the following equations

$$\dot{\Psi} = \frac{3\mu}{\omega_z r_s^3} \frac{I_{zz} - \dfrac{I_{xx}+I_{yy}}{2}}{I_{zz}} \cos\theta\cos^2\phi \tag{13}$$

$$\dot{\eta} = \frac{3\mu}{\omega_z r_s^3} \frac{I_{zz} - \dfrac{I_{xx}+I_{yy}}{2}}{I_{zz}} \cos\theta\sin\phi\cos\phi \tag{14}$$

The average precession over one complete orbit of Eros about the Sun is given by

$$\dot{\Psi}_{avg} = \frac{3\mu}{\omega_z a_s^3} \frac{I_{zz} - \dfrac{I_{xx}+I_{yy}}{2}}{I_{zz}} \cos\theta \, \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos^2\phi}{(1-e\cos\phi)^3}\, d\phi \tag{15}$$

where

$$r_s = a_s(1 - e\cos\phi)$$

and, for simplicity, the projection of the Eros pole on the orbit plane is in the direction of periapsis. After carrying out the integration we obtain,

$$\dot{\Psi}_{avg} = \frac{3\mu}{2\omega_z a_s^3} \frac{I_{zz} - \dfrac{I_{xx}+I_{yy}}{2}}{I_{zz}} \cos\theta\,(1-e^2)^{-\frac{3}{2}} \tag{16}$$

Observe that the moments of inertia are related to the gravity harmonics by

$$I_{zz} - \frac{I_{xx}+I_{yy}}{2} = -mr_0^2 C_{20} \tag{17}$$

where m is the mass of Eros, r_0 is the reference radius used for computing gravity harmonic coefficients and C_{20} is the gravity harmonic coefficient related to oblateness. Solving for the moment of inertia about the z axis we obtain an equation for I_{zz}.

$$I_{zz} = \frac{3\mu}{2\omega_z a_s^3} \frac{-mr_0^2 C_{20}}{\dot{\Psi}_{avg}} \cos\theta\,(1-e^2)^{-\frac{3}{2}} \tag{18}$$

The above equation gives I_{zz} in terms of quantities that are directly observable. The most difficult parameter to observe is $\dot{\Psi}_{avg}$. If observed over many years, a direct measure of I_{zz} is obtained. The average nutation integrates to zero over one orbit of Eros around the Sun. An equation for a moving average is given by

$$\dot{\eta}_{avg} = \frac{3\mu}{\omega_z a_s^3} \frac{-mr_0^2 C_{20}}{I_{zz}} \cos\theta \; \frac{1}{\phi} \int_0^{\phi} \frac{\sin(\phi)\cos(\phi)}{(1 - e\cos\phi)^3} \, d\phi \tag{19}$$

For simplicity we have assumed that the projection of the Eros spin axis on its orbit plane is in the direction of periapsis and the orbit is circular. Carrying out the indicated integration we obtain

$$\dot{\eta}_{avg} = \frac{1}{\phi} \frac{3\mu}{2\omega_z a_s^3} \frac{-mr_0^2 C_{20}}{I_{zz}} \cos\theta \; \sin^2\phi \tag{20}$$

The above equations for precession and nutation describe the direction of the Eros pole in inertial space as a function of time. With the simplifying assumptions made to obtain an analytic solution, the equations actually describe the direction of the angular momentum vector. If the solar gravity gradient is the only attitude perturber, the pole and angular momentum vector very nearly coincide. This gives rise to what is termed forced precession where the migration of the pole is dependent primarily on the external forces and not the inertial properties of the body. Free precession and nutation, that occurs when the spin axis and angular momentum vector are not aligned, is the type of motion that is generally associated with rotating bodies and is primarily dependent on the inertial properties of the body. For this reason, more can be determined about the inertial properties of Eros if free precession can be observed and measured. The magnitude of the free precession that may be attributed to the solar gravity gradient is extremely small. However, it is not zero because the direction of the angular momentum vector cannot be changed without inducing some free precession in the body.

Figure 7 shows the free precession of the Eros pole that would be observed if there was no solar gravity gradient acting on Eros. The initial attitude rates are set equal to the rates that would be induced by the solar gravity gradient acting over some time. The pole of Eros processes and nuttiness about the angular momentum vector. The precession is the large elliptical shaped curves and the notation is the small closed curves. Since the maximum amplitude is only 1 anno radian, this motion is far below our current capabilities to detect. Figure 8 shows the short term forced precession that occurs as a result of the solar gravity gradient torque. The time duration is about 30 hours and includes just under 6 revolutions of Eros. The magnitude of the attitude motion over this time interval is about one micro radian which is still below our current capabilities to detect. The small wiggles in the curve are the result of the free precession. Figure 9 shows the forced precession over three and one half years or two complete orbits of Eros about the Sun. This curve shows the characteristic grand precession and notation that are familiar to astronomers. At the scale shown here, the free precession and notation cannot be discerned. Over this time span, the 0.2 milli radian change corresponds to 3 meters measured at the end of Eros. With current optical navigation techniques, it may be possible to detect the forced precession over the duration of the mission. If Eros rotation could be observed on a mission that returns to Eros in thirty years, a measurement of the precession would yield a determination of the moment of inertia about the spin axis that would be useful for determining the internal mass distribution of Eros. In any event, the model of Eros rotational motion should include the solar gravity gradient.

Shape Model Harmonic Coefficients

The shape model determined from Eros images by Peter Thomas at Cornell has been transformed to a triangular plate model for use by the NEAR project. A shape model is needed for processing the NEAR Laser Rangefinder (NLR) data which will be used as a backup data type for navigation. The NLR measures the distance from the spacecraft to a point on the surface of Eros by timing the transmission and reception of a laser light pulse that is reflected from the surface. In order to be useful for navigation, the range from the spacecraft to the center of Eros must be determined. This is achieved by adding the vector from the center of Eros to the surface point to the vector from the surface point to the spacecraft. The first of these vectors is determined from a shape model of Eros

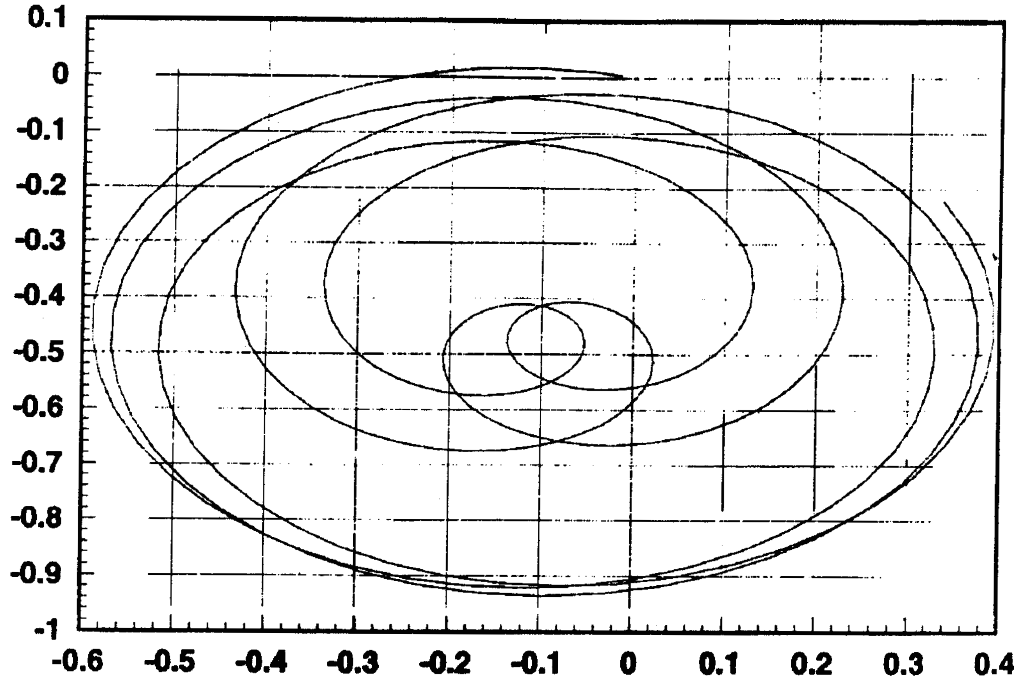

Figure 7 Eros Free Precession and Nutation - nanoradians

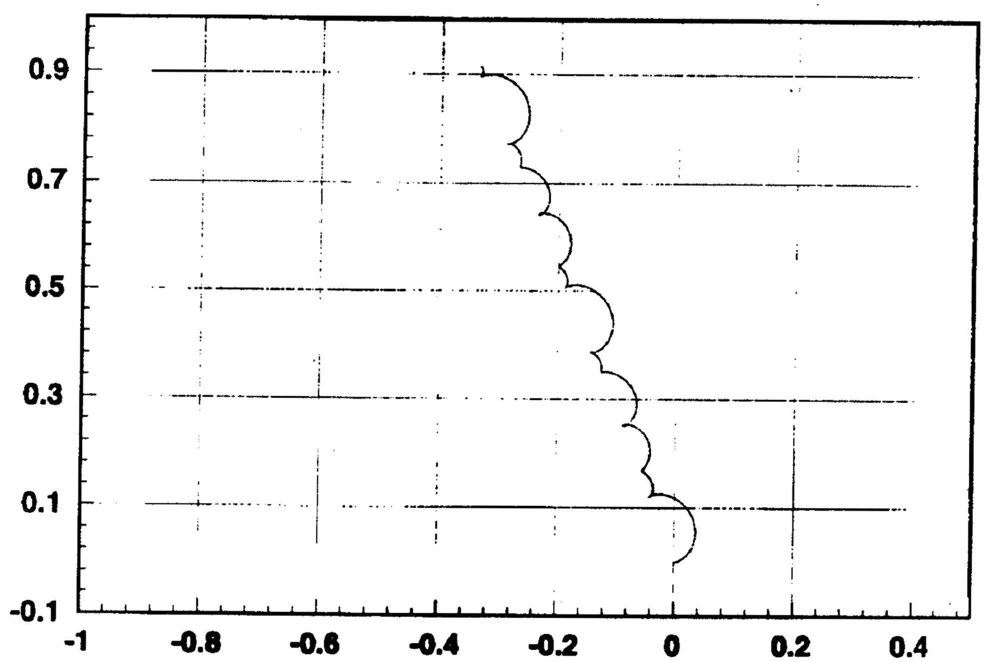

Figure 8 Eros Short Term Forced Precession and Nutation - microradians

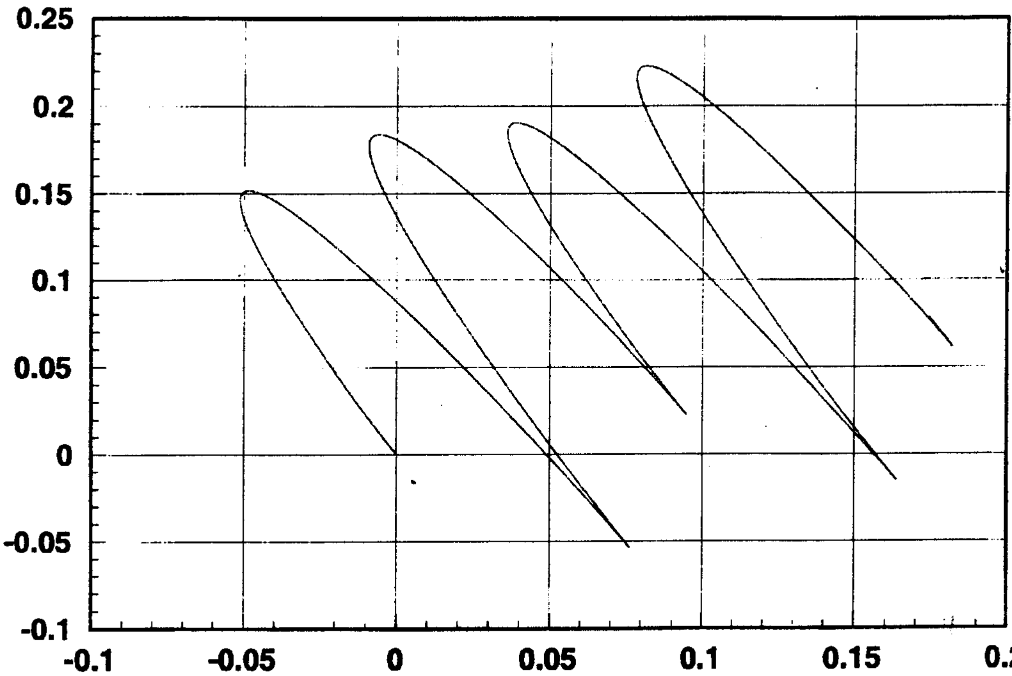

Figure 9 Eros Long Term Forced Precession and Nutation - milliradians

and the second from the measured distance and attitude of the NLR instrument when the pulse is transmitted and received. Implementation of the NLR data type in the orbit determination filter requires computation of the partial derivatives of the measurement with respect to spacecraft state, Eros attitude and parameters of the Eros shape model. The triangular plate model contains sharp edges at the boundary of the plates and as a result the partial derivatives are discontinuous. We need a shape model that will provide continuous partial derivatives for the filter to converge a solution. The implementation used for NEAR navigation employs a shape model consisting of Legendre polynomial and associated functions harmonic coefficients. A further benefit of a harmonic coefficient model is that the surface normal vector may be computed easily from the gradient and subroutines that have been thoroughly checked out for gravity harmonics may be used in the software implementation. The orbit determination solution includes spacecraft state as well as the harmonic coefficients. The orbit determination filter must be initialized with *a priori* harmonic coefficients and these may be computed from the triangular plate model. There are two methods that have been implemented for computing the shape harmonic coefficients from the triangular plate model. The first involves direct numerical integration of the coefficient generating functions over the surface defined by the plate model. The second involves performing a least square fit of many values of the radius sampled uniformly over the entire surface to the harmonic coefficients. Intuitively it would seem advantageous to perform the numerical integration and determine the shape harmonic coefficients directly. The least square method involves inversion of a matrix that is of a dimension equal to the number of coefficients that number 2601 for a 50 degree and order expansion. However, the harmonic coefficients happen to be linear in terms of the radius and only one inversion is required. The least square fit method appears to yield better results. Figure 10 shows several simulated images of Eros from the models as viewed looking down on the north pole. The plate model provides a frame of reference for comparison. Images are shown for harmonic expansions of degree 8, 16 and 50. The 16 degree expansion provides a fairly close approximation to the plate model and involves solving for only 289 coefficients. The 50 degree expansion looks identical to the plate model. Some of the angular surfaces are rounded. If a high precision shape model is required, the solution for 2601 coefficients is within the capability of existing software which uses square root filtering.

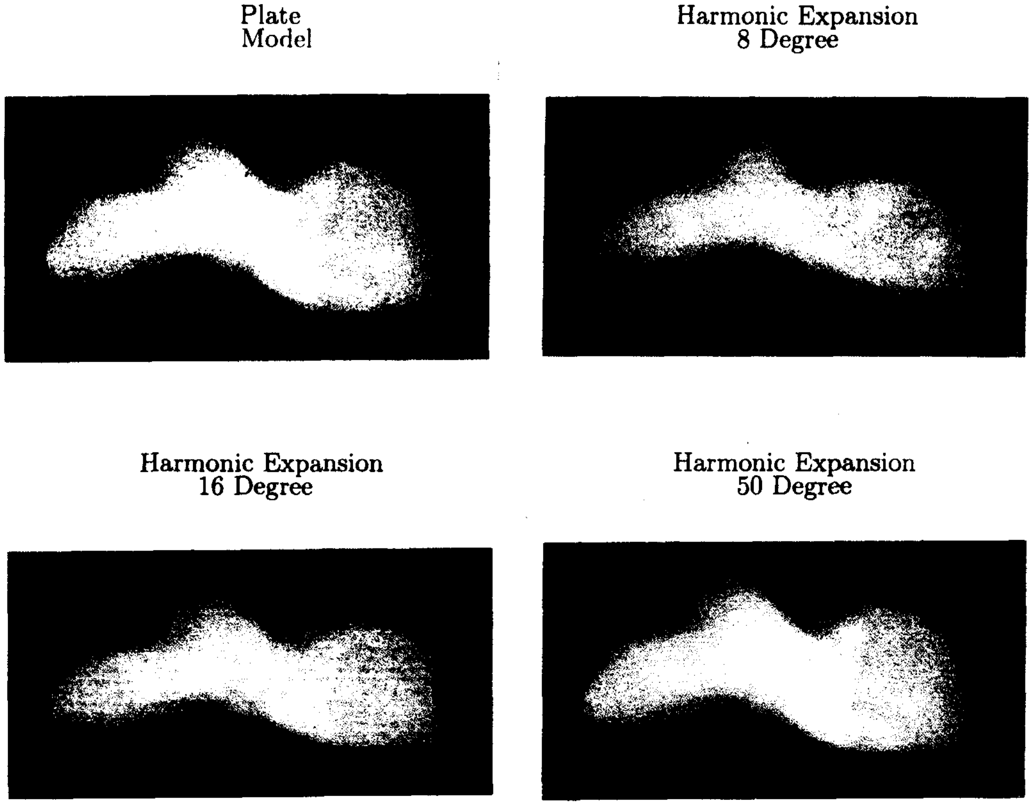

Figure 10 Comparison of Plate Model with Harmonic Expansion Models

Gravity Harmonics from Shape Model

Gravity harmonic coefficients of Eros are an output of the orbit determination process. The harmonic coefficients are estimated by observing the acceleration of the spacecraft in orbit about Eros. As the spacecraft is maneuvered closer to Eros the degree of the harmonic expansion is increased in order to provide the required accuracy for orbit prediction. During the flyby of Eros, the gravity field was barely detected and no determination was attempted for the harmonic coefficients. If the density of Eros is uniform, the harmonic coefficients may be computed from the observed shape. The assumption of constant density as a basis for computing gravity harmonics would not be wise once in orbit about Eros since it would almost surely lead to an erroneous computation of spacecraft acceleration. However, the constant density assumption would probably provide a gravity model that is good for designing the orbit phase trajectory and would serve as *a priori* for the in orbit solution. Since the shape of Eros is the only information we have that may be used to infer a gravity field as we approach Eros, it seems prudent to make use of this information.

The gravity harmonic coefficients may be determined by numerically integrating the coefficient generating functions over the surface of Eros using the triangular plate model developed by Cornell. This model gives the radius as a function of latitude and longitude and this is all that is needed to compute the harmonic coefficients. The resultant harmonic expansion is of degree and order 16 and is referred to as the reference gravity model. Some of the harmonic coefficients that were determined by this method are given above in Table 2. A problem with the reference gravity model is expected when it is needed to compute spacecraft acceleration very close to Eros. This will occur late in the mission when the spacecraft is maneuvered for some close observations of Eros. When the spacecraft descends below a radii that is less than the maximum radii of Eros (about 16 km) the harmonic expansion diverges. At 5 km altitude over the pole, the error in computing spacecraft acceleration is so great as to render the reference model useless. Some alternative models are being pursued for this phase of the mission. One approach that appears promising is to subdivide Eros into many small pieces and compute the acceleration of the spacecraft by adding up the acceleration from each piece. This method will work provided the spacecraft stays outside the radii of convergence for each piece. A plot of the acceleration magnitude as a function of geocentric altitude above the surface of Eros for a 5 km altitude is shown on Figure 11. For this gravity model, Eros was subdivided into 12 roughly equal pieces and a fourth degree gravity field was computed for each piece. Since a fourth degree expansion requires 25 harmonic coefficients, the total number of coefficients was 300 (12 times 25). This number of coefficients is well within the solution capability of the orbit determination filter. The acceleration surface reveals minimums above the ends of Eros that are consistent with the surface computed at higher altitudes from the reference model.

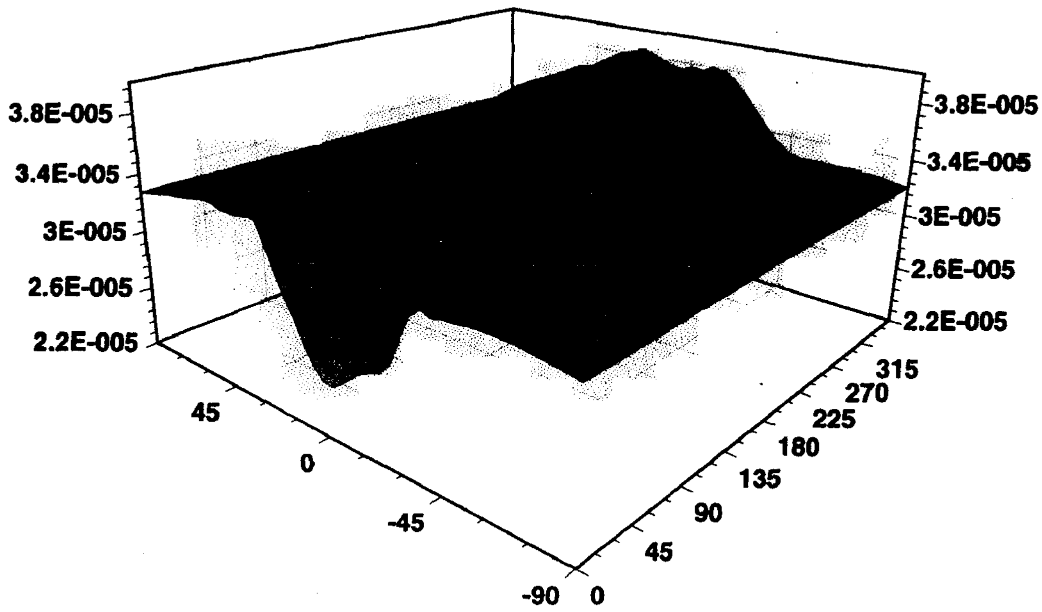

Figure 11 Acceleration of Point mass at 5 km Altitude - km/s^2

SUMMARY

The unplanned flyby of Eros on December 23, 1998 at a distance of 3830 km provided an unusual oportunity to obtain quantitative measurements of Eros physical parameters as well as a qualitative assessment of the adequacy of the navigation system to navigate the NEAR spacecraft in orbit about Eros. From the viewpoint of mission design, the most important parameters to determine are the mass of Eros and the direction of the pole or spin axis. The mass of Eros is $(7.2 \pm 1.8) \times 10^{18}$ grams and the pole position is $15.6(\pm 3.7)$ degrees in right ascension and $16.4(\pm 1.8)$ degrees in declination. These results are consistent with ground based observations and place a bound on these parameters that reduces the amount of trajectory design options that must be studied in preparation for the February 2000 Eros orbit phase.

Another benefit is the determination of *a priori* values for the parameters that are required to be estimated by the orbit determination filter. Harmonic coefficients for the gravity model were determined from the Eros shape model developed from images of Eros taken during the flyby. In addition, harmonic coefficients were determined for the shape model to be used for processing NLR data. The advantage of having good *a priori* values for parameters is reduced number of iterations required to converge orbit determination solutions and thus timely delivery of spacecraft trajectory information to the project once in orbit. From a qualitative assessment of the images obtained during the flyby, the unique shape characteristics of Eros will permit positive identification of landmarks by reference to the large scale surface features. A more benign shape for Eros could have made the landmark identification process difficult. The delay in the onset of Eros orbital operations was certainly a disappointment for members of the NEAR flight operations team. However, from the viewpoint of the Navigation Team, this may have been a fortuitous occurrence in that we will enter Eros Orbit operations much better prepared.

ACKNOWLEDGMENT

The work described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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